The Definition of the Derivative

The derivative of f at $x = a$ is defined to be the following (both limits are equivalent):

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Often we are given a limit that represents the derivative of some function at a value and need to find f and a. The following is a process to identify that information.

Identifying Information from Limits

How to identify f and a

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

1. Identify a
   - Look for something in the form $a + h$
2. Identify the function
   - Replace $a + h$ with x
   - What is happening to x?

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

1. Identify a
   - This is easy. Look at the denominator. It is in the form $x - a$, so whatever is being subtracted from x is a
2. Identify $f(x)$
   - The numerator is in the form $f(x) - f(a)$, so this should be pretty clear as well