Work

Things to Know About Work

$$W = Fd \qquad W\text{: Work}$$
$$F = ma \qquad F\text{: Force}$$
$$m = \delta V \qquad m\text{: mass}$$

a: acceleration (gravity)

δ: density

$$\implies W = \delta V a d \qquad V\text{: volume}$$

- When units are in m and kg, $a = 9.8$ m/s^2
- When units are in ft and lb, δa is a single quantity known as "weight-density", and can be calculated based on the information in the problem. The weight density of water is 62.5 lb/ft^3, and the mass of water is 1000 kg/m^3
- Force is measured in lbs or Newtons

Solving Spring Problems

The *force* required to hold a spring x distance beyond its natural length is given by

$$F = kx$$

The *work* required to move a spring from a to b beyond its natural length is given by

$$W = \int_a^b kx\,dx$$

1. Convert all numbers to the proper units
   - Units should be meters and Newtons, or pounds and feet
2. Using the given information, find the spring constant k
3. Plug this into the equation for the desired quantity

Remember!

a and b are measure in distance **beyond the natural length**!

Solving Lifting Problems

Assuming that what you are lifting has uniform density, and you are lifting at a constant rate:

1. Determine what x will represent (the length you have pulled up, the length *not* yet pulled up, etc.)
   - You can choose anything you want for x, but remember to **be consistent** after you'd made your choice.
2. Determine two known points on your force function
   - Usually you know the starting and ending forces
3. Find the $F(x)$, the function that gives the force being applied given any x. This function will be linear, given the conditions of uniform density and constant lifting rate.
4. Determine the bounds, a and b, on the integral. These are the values of x for which you are actually applying a force.
5. Integrate $\int_a^b F(x)dx$

Solving Pumping Problems

1. Set up a coordinate axis system on the tank. Be sure to place it so that the variable edges appear on the xy-plane
2. Draw a horizontal slice of fluid.
3. Determine the bounds - the values of y at which we have fluid that we need to move
4. Determine δa
   - Assuming the fluid being moved is water, if units are in feet/pounds: $\delta a = 62.5$
   - If units are in meters/kg: $\delta a = 9800$
5. Identify the vertical level that the fluid is being pumped to, $y = \ell$
6. The distance traveled for each slice is $\ell - y$
7. Find the volume V of the slice, determined by the geometric shape of the tank. This includes a dy
8. Integrate $\int_a^b \delta V a(\ell - y)$