

## The Definition of a Limit

We say  $\lim_{x \rightarrow a} f(x) = L$  if for every  $\epsilon > 0$ , there exists a  $\delta > 0$  such that

$$0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$$

## How to Solve $\epsilon - \delta$ Problems

Given  $\epsilon > 0$ , do the following:

1. Set  $f(x) = L + \epsilon$  and  $f(x) = L - \epsilon$
2. Solve for  $x$
3. Compute  $|x - a|$  for each  $x$
4. The smallest result is  $\delta$

This method will work for any function, but Step 2 may require a calculator.

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If not given  $\epsilon$ , do the following

1. Set up  $|f(x) - L| < \epsilon$
2. Use algebra to get  $|x - a|$  on the left
3. The quantity on the right (a function of  $\epsilon$ ) is  $\delta$

Generally, this method will only be useful for linear functions. When  $f(x)$  is linear, you can use this even when you know  $\epsilon$ .