

Types of Discontinuities

Infinite Discontinuity

We say that $f(x)$ has an infinite discontinuity at $x = a$ if

- $\lim_{x \rightarrow a^+} f(x) = \pm\infty$
- or**
- $\lim_{x \rightarrow a^-} f(x) = \pm\infty$

Jump Discontinuity

We say that $f(x)$ has a jump discontinuity at $x = a$ if

- $\lim_{x \rightarrow a^-} f(x) \neq \lim_{x \rightarrow a^+} f(x)$
- and**
- Both limits are finite

Removable Discontinuity

We say that $f(x)$ has a removable discontinuity at $x = a$ if

- $\lim_{x \rightarrow a} f(x)$ exists
- and**
- $\lim_{x \rightarrow a} f(x) \neq f(a)$

Checking for Discontinuity

Evaluate the one-sided limits:

$$\lim_{x \rightarrow a^-} f(x) \text{ and } \lim_{x \rightarrow a^+} f(x)$$

Are they both finite?

- No: there is an **infinite discontinuity** at $x = a$
- Yes: go to the next step

Compare the one-sided limits

Are the one-sided limits equal?

- No: there is a **jump discontinuity** at $x = a$
- Yes: go to the next step

Evaluate $f(a)$

Does $\lim_{x \rightarrow a} f(x) = f(a)$?

- No: there is a **removable discontinuity** at $x = a$.
- Yes: f is continuous at a