Continuity: Examples

SYMPLIT

1. Sketch the graph of a function this is continuous except for the following:
   - Jump discontinuity at -2
   - Infinite discontinuity at 1
   - Removable discontinuity at 3
2. Sketch the graph of a function this is continuous except for the following:
   - Discontinuous from the left, but continuous from that right at -3
   - Discontinuous from the left and right at -1
   - Continuous only from the right at 2
3. Let $C(x)$ be the function
$$C(x) = \begin{cases} \ln(-x-3) & \text{if } x < -3 \\ |1-x| & \text{if } -3 \le x < 2 \\ 4 & \text{if } x = 2 \\ 2\sqrt{2x} & \text{if } 2 < x \le 8 \\ 8 & \text{if } x > 8 \end{cases}$$
   (i) T/F: $\lim_{x \to -3^+} C(x) = C(-3)$
   (ii) T/F: $C(x)$ has a removable discontinuity at $x = -3$
   (iii) T/F: $\lim_{x \to 2^-} C(x) = C(2)$
   (iv) T/F: $C(8) = 8$
   (v) T/F: $\lim_{x \to 8} C(x)$ exists
   (vi) T/F: $C(x)$ has a jump discontinuity at $x = 8$ d
4. Let $f(x) = \begin{cases} 3x - c & \text{if } x < 0 \\ e^x & \text{if } x \ge 0 \end{cases}$
   What value of c makes $f(x)$ continuous?
5. Find c such that the following function is continuous:
$$f(x) = \begin{cases} x^3 + cx + 3 & \text{if } x \le 2 \\ 4x + c & \text{if } x > 2 \end{cases}$$
6. How would you remove that discontinuity of $f(x) = \dfrac{1-x}{x^2-1}$? In other words, how would you define $f(1)$ in order to make f continuous at 1?
7. Show that there is a root of the equation $x^3 + 3x + 1 = 0$ on the interval $(-1, 0)$
8. Prove that the equation $\sin x + x^5 = 3$ has at least one real root.