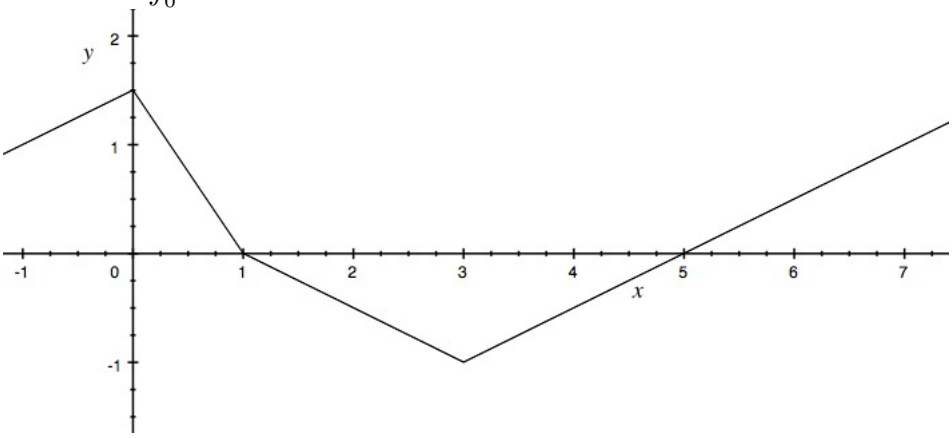
Fundamental Theorem of Calculus: Examples

SYMPLIT

1. Let $g(x) = \int_0^x f(t)dt$ where there graph of f is shown.



   (a) Evaluate $g(0)$, $g(1)$, $g(3)$, $g(5)$, and $g(7)$

   (b) On what interval(s) is g increasing and decreasing?

   (c) Where does g have extrema?

   (d) Sketch the graph of g.

2. Differentiate the function: $g(r) = \int_3^r (t^2 + \sin t)^5 dt$

3. Differentiate the function: $P(t) = \int_t^5 \sqrt{u \ln u}\, du$

4. Differentiate the function: $y = \int_4^{\sin x} (1 - t^2)^3 dt$

5. Evaluate the integral: $\int_1^2 (3 - x + 4x^3) dx$

6. Evaluate the integral: $\int_1^3 \dfrac{2 - x^2}{x^3} dx$

7. Evaluate the integral: $\int_{\pi/3}^{2\pi/3} \csc^2 x\, dx$

8. Evaluate the integral: $\int_0^2 f(x)dx$ where $f(x) = \begin{cases} 1 - x^2 & \text{if } 0 \le x < 1 \\ x^2 - 2x & \text{if } x \ge 1 \end{cases}$

9. What's wrong with this equation? $\int_{\pi/4}^{3\pi/4} \sec^2 x\, dx = \tan x \Big|_{\pi/4}^{3\pi/4} = -2$

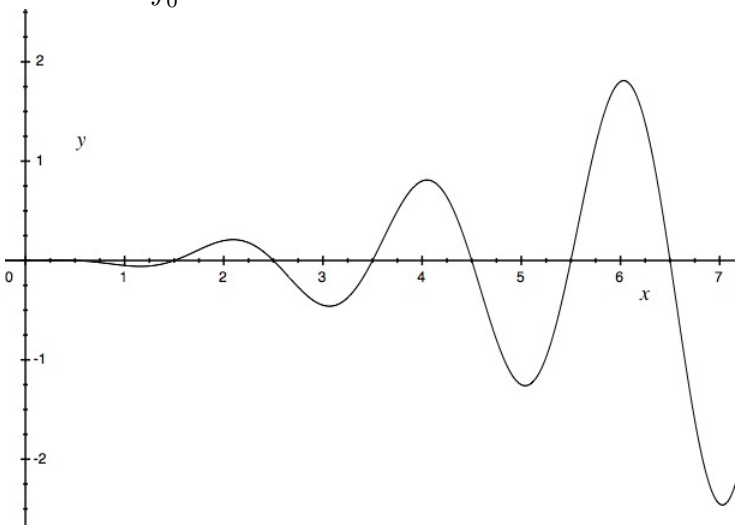
10. Differentiate the function: $h(x) = \int_{\sqrt{x}}^{4x+1} \left(\dfrac{u}{u^2 - 1} \right) du$

11. Differentiate the function: $g(x) = \int_x^{\sqrt[3]{x}} \sin(t^3)dt$

12. If $f(x) = \int_0^{\ln x} \sqrt{t}\, dt$ and $g(y) = \int_4^y f(x)dx$, find $g''(e)$.

13. If $g(2) = 13$, g' is continuous, and $\int_{-3}^{2} g'(x)dx = 5$, find $g(-3)$.

14. Let $g(x) = \int_0^x f(t)dt$, with the graph of f shown below



(a) Where does g attain its local maximum and minimum values?

(b) On $[0, 7]$, where does g attain its absolute maximum value?

(c) On what intervals is g concave up and concave down?

(d) Sketch the graph of g

15. Evaluate the limit by recognizing it as a Riemann sum: $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4i^3}{n^4} - \frac{2}{3n}\sqrt{\frac{i}{n}} \right)$

16. Evaluate the limit by recognizing it as a Riemann sum: $\lim_{n\to\infty} \frac{2}{n} \left(\left(\frac{2}{n}\right)^2 + \left(\frac{4}{n}\right)^2 + \cdots \left(\frac{2n}{n}\right)^2 \right)$

17. Below is the graph of $y = x^2$. If the area of region B is 4 times the area of region A, express b in terms of a