

1. A metal cylindrical container with an open top is to hold one cubic foot. Find the dimensions that require the least amount of material.
2. A page of a book is to have an area of 90 in^2 , with 1-inch margins at the bottom and sides and a $1/2$ -inch margin at the top. Find the dimensions of the page that will allow the largest printed area.
3. Find the point on the graph of $y = x^2 + 1$ that is closest to the point $(3, 1)$.
4. Find the dimensions of the rectangle of maximum area that can be inscribed in a semicircle of radius 5, if two vertices lie on the diameter.
5. A pipeline for transporting oil will connect two points A and B that are 7 miles apart and on the opposite banks of a straight river 2 miles wide. Part of the pipeline will run under water from A to a point C on the opposite bank, and then above ground from C to B . If the cost per mile of running the pipeline under water is four times the cost per mile of running it above ground, find the location of C that will minimize the cost.
6. Two corridors 3 feet and 4 feet wide, respectively, meet at a right angle. Find the length of the longest nonbendable rod that can be carried horizontally around the corner.
7. At 1:00 PM ship A is 30 miles due south of ship B and is sailing north at a rate of 15 mph. If ship B is sailing west at a rate of 10 mph, find the time at which the distance d between the ships is minimal.
8. A window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the dimensions that will allow the maximum amount of light to enter.
9. A water cup in the shape of a circular cone is to be constructed by removing a circular sector from a circular sheet of paper of radius a and then joining the two straight edges of the remaining paper. Find the volume of the largest cup that can be constructed.
10. Find the largest possible volume for a cylinder inscribed in a sphere of radius r .