Mean Value Theorem: Examples

1. Find the value c that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^2 + 3x - 1$ on $[0, 2]$.

2. Find the value of c that satisfies the conclusion of the Mean Value Theorem for $f(x) = e^{2x}$ on $[-1, 1]$

3. Let $f(x) = \frac{1}{x-1}$. Show that there is no value of c in $(0, 2)$ such that $f'(c) = \frac{f(2)-f(0)}{2-0}$. Why does this not contradict the Mean Value Theorem?

4. Let $f(x) = 3 + |x - 2|$. Show that there is no value of c in $(0, 5)$ such that $f'(c) = \frac{f(5)-f(0)}{5-0}$. Why does this not contradict the Mean Value Theorem?

5. Show that the equation $e^{2x} + 3x = 0$ has exactly one real root.

6. Show that the equation $\sin(3x) - 7x = 0$ has exactly one real root.

7. Suppose f is an even function and is differentiable everywhere. Prove that $f'(0) = 0$.

8. Use the Mean Value Theorem to prove that $|\cos a - \cos b| \leq |a - b|$.

9. Let f be continuous and differentiable everywhere. If $f(2) = 5$ and $f'(x) \geq -2$, what is the smallest value that $f(10)$ can have?

10. Let f be continuous and differentiable everywhere. If $f(-3) = 7$ and $-3 \leq f'(x) \leq 4$, find values a and b such that $a \leq f(25) \leq b$.