Calculus with Parametric Curves: Examples

SYMPLIT

1. A curve is parametrized by the equations $\begin{cases} x = 6\sin t \\ y = t^2 + t \end{cases}$. Find the slope of the line that is tangent to this curve at the point $(0, 0)$.

2. Find the equation of the tangent line to the curve $\begin{cases} x(\theta) = \theta - \sin\theta \\ y(\theta) = 1 - \cos\theta \end{cases}$ when $\theta = \pi/3$

3. Find the equation of the tangent line to the curve $\begin{cases} x = e^t \\ y = t^2 \end{cases}$ when $t = 0$

4. Find all points (if any) where the curve $\begin{cases} x = 6\sin t \\ y = t^2 + t \end{cases}$ has a horizontal tangent

5. Find all points (if any) where the curve $\begin{cases} x(\theta) = \theta - \sin\theta \\ y(\theta) = 1 - \cos\theta \end{cases}$ has a horizontal tangent

6. Find all points (if any) where the curve $\begin{cases} x = e^t \\ y = t^2 \end{cases}$ has a vertical tangent

7. Set up the integral used to find the length of the curve defined by the parametric equations

$$\begin{aligned} x &= e^t - t \\ y &= 4e^{t/2} \qquad 0 \le t \le 1 \end{aligned}$$

8. The position, in feet, of a slow pitch softball at time t, in seconds, is given by the parametric equations $\begin{cases} x = 18\sqrt{3}t \\ y = -16t^2 + 18t + 4 \end{cases}$.

   (a) What is the rate of change of the height of the ball with respect to its horizontal position when it crosses the plate at $t = 1.2$ seconds?

   (b) Set up an integral giving the arc length of the path of the ball for $0 \le t \le 1.2$

9. Set up the integral used to find the length of the curve defined by the parametric equations

$$\begin{cases} x(\theta) = \theta - \sin\theta \\ y(\theta) = 1 - \cos\theta \end{cases}, \qquad 0 \le \theta \le 2\pi$$

10. Find the area above the x-axis and under the curve $\begin{cases} x(\theta) = \theta - \sin\theta \\ y(\theta) = 1 - \cos\theta \end{cases}$, $0 \le \theta \le 2\pi$